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EPQ Model for Deteriorating products with demand dependent production rate under Advance-Cash-Credit Payment scheme

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Abstract: In today's modern business transactions, the supplier of raw meterials usually asks the manufacturer to pay via the advance-cash-credit (ACC) payment policy. According to this ACC payment policy, the manufacturer pays a fraction of the procurement cost as advance payment when signing a contract to buy raw meterials, pays another fraction of the procurement cost as cash payment in cash when receiving the ordered quantity, and receives a short-term interest-free credit period to pay the remaining procurement cost as credit payment. In this article, we develop an EPQ model for deteriorating products under the ACC payment policy.The main goal of this article is to determine an optimal selling price while maximizing the total profit of the manufacturer. In addition, some numerical examples are provided to illustrate the developed EPQ model and show the concavity of the profit function with respect to selling price.

Keywords: *EPQ model, Advance-cash-credit (ACC) payment, and Deteriorating products.*

1. Introduction and Literature Review

The traditional economic production quantity (EPQ) and economic order quantity (EOQ) models do not investigate payment methods to increase sales and market share and tacitly assume that the payment is made immediately upon receiving the consignment of goods. However, in real world business transactions, an advance-cash-credit (ACC) payment scheme is commonly used to develop EOQ/EPQ models. When the purchase cost is very large, to avoid customer defaults and to reduce holding stock, the manufacturer usually agrees with the wholesaler on the following payment method. The wholesaler is required to prepay a fraction of the procurement cost as a contract to buy products, then pay another fraction of the procurement cost in cash upon receiving the ordered quantity and receive a short interest-free credit period to pay the remaining fraction of the procurement cost. This payment method is known as an advance-cash-credit (ACC) payment scheme.

Goyal (1985) discussed the economic order quantity under conditions of permissible delay in payments. An implicit assumption of Goyal's model is that the items are obtained from an outside supplier and the entire lot size is delivered at the same time. Chung and Huang (2003) extended Goyal's model and formulated an EPQ model under supplier's trade credit policy. Mahata (2012) formulated an EPQ model for deteriorating items with up-stream full trade credit and down-stream partial trade credit. He also assumed that the retailer maintains a powerful decision-making right and can obtain the full trade credit offered by the supplier yet retailer just offers the partial trade credit to its customers. Feng et al. (2013) investigated the retailer's optimal cycle time and optimal payment time under the supplier's cash discount and trade credit policy within the economic production quantity (EPQ) framework. They assumed that the retailer will provide a full trade credit to its good credit customers and request its bad credit customers pay for the items as soon as receiving them. Chen et al. (2014) proposed an EPQ model for deteriorating items in a supply chain with both up-stream and down-stream trade credit financing. By using fractional programming results, they prove that the optimal solution not only exists but also is unique.

Chakraborty et al. (2016) formulated an environment friendly economic production quantity (EPQ) model for a single product under trade credit. The trade credit offered by the raw material supplier depends on the amount of raw material purchased through some fuzzy rules. Taleizadeh et al. (2016) developed an imperfect EPQ model with upstream trade credit periods linked to raw material order quantity and downstream trade credit periods. Wu et al. (2018) presented an inventory model for perishable products with expiration date dependent deterioration under an advance-cash-credit payment scheme to find the optimal replenishment cycle time such that the total profit is maximized. Tsao et al. (2019) developed an EPQ model for perishable products under the advance-cash-credit payment scheme with a discounted cash flow analysis. Li et al. (2019) developed an inventory model interfaced with marketing, operations, and finance in a supplier-retailer chain in which the demand curve is downward sloping. They demonstrated that an increase in the fraction of advance payment raises selling price, while an increase in the fraction of credit payment reduces selling price. Udayakumar et al. (2021) framed an inventory model for non-instantaneous deteriorating items by considering money inflation and time discounting, where a permissible delay period is offered by the supplier as an alternative to price discount.

In this paper, an EPQ model for deteriorating items under advance-cash-credit payment scheme is developed. The complete paper is arranged as follow. In section 2, assumptions and notations are explained and in section 3, the EPQ model is formulated mathematically to find the manufacturer's profit function. In section 4, numerical analysis are performed with the help of the examples to find the numerical solution of the EPQ model. Finally, the conclusion of the model and suggestions for future research scope are provided in section 5.

2. Assumptions and Notations

To develop the proposed production inventory model, the following assumptions and notations are made.

1. The proposed inventory model is developed for a single product.
2. The products considered here are deteriorating in nature and the deteriorated products are not replaced and repaired during the replenishment cycle.
3. There is a single supplier of the raw materials and a single manufacturer of the final product.
4. The demand rate of the final product is a decreasing function of selling price and is taken as $D(p)=a-bp$, where $a,b \geq 0$.
5. The production rate depends on the demand of the final product and is taken as $P=\lambda D(p)=\lambda(a-bp)$, where $\lambda>1$.
6. Replenishment is instantaneous and shortages are not allowed. The time horizon is infinite.
7. The supplier of the raw materials makes an advance-cash-credit (ACC) payment agreement with the manufacturer. According to this ACC payment agreement:
   a) The manufacturer pays α fraction of procurement cost as advance payments at the time when order is placed by the manufacturer.
   b) The manufacturer pays β fraction of procurement cost as cash payments at the time when the placed order is received by the manufacturer.
   c) The manufacturer receives a permissible credit period of M years to pay the remaining τ portion of procurement cost as credit payments.
8. The manufacturer takes a loan from a financial institution to pay advance and cash payments for the procurement cost. The financial institution imposes the condition that the manufacturer will pay loan with interest at the end of replineshment cycle.

Notations	Description
a, b	: Demand parameters
θ	: Deterioration parameter with $0 \leq \theta << 1$
λ	: Production parameter with $\lambda > 1$
α	: Fraction of procurement cost to be paid as advance payments by the manufacturer before the time of delivery, $0 \leq \alpha \leq 1$
β	: Fraction of procurement cost to be paid as cash payments by the manufacturer at the time of delivery, $0 \leq \beta \leq 1$
τ	: Fraction of procurement cost granted a credit period $[0, M]$ from the supplier to the manufacturer, $0 \leq \tau \leq 1$ and $\alpha+\beta+\tau=1$
M	: Length of credit period offered by the supplier to the manufacturer
L	: Length of time during which the manufacturer will pay the advance payments, where $L>0$
v	: Time at which the production is stoped by the manufacturer

O	: Ordering cost per order
h	: Holding cost per unit per unit time
c	: Procurement cost per unit
p	: Selling price of the item per unit
d	: Deterioration cost per unit per unit time
$\mathrm{I}_1(\mathrm{t})$	: Inventory level during the time interval $[0,v]$
$\mathrm{I}_2(\mathrm{t})$	: Inventory level during the time interval $[v,T]$
A	: Total money in interest bearing account at the time $\mathrm{t}=\mathrm{T}$
T	: Length of inventory cycle time
I_e	: Rate of interest earned by the manufacturer
I_s	: Rate of interest charged by the supplier of raw materials with $I_e \le I_s \le I_c$
I_c	: Rate of interest charged by the financial institution with $I_c \ge I_e$

3. Mathematical Formulation of the EPQ Model

In this section, a mathematical model is formulated to describe the economic production quantity (EPQ) model under advance-cash-credit (ACC) payment methods. The manufacturer places an order of raw materials for production to the suppliers of raw materials at time $\mathrm{t}=-\mathrm{L}$ and receives it from the suppliers at time $\mathrm{t}=0$. After receiving the raw materials, the manufacturer starts the production at time $\mathrm{t}=0$. The manufacturer accumulates continuously the finished products in the warehouse after adjusting demands and deterioration during the production period $[0,v]$. The manufacturer will stop the production at time $t=v$. The accumulated stock is sufficient enough to adjust the demands and deterioration over the interval $[v,T]$. Subsequently, the inventory cycle ends with zero stock at time $t=T$. The entire process repeats itself. The above defined production model is shown in the figure 1.

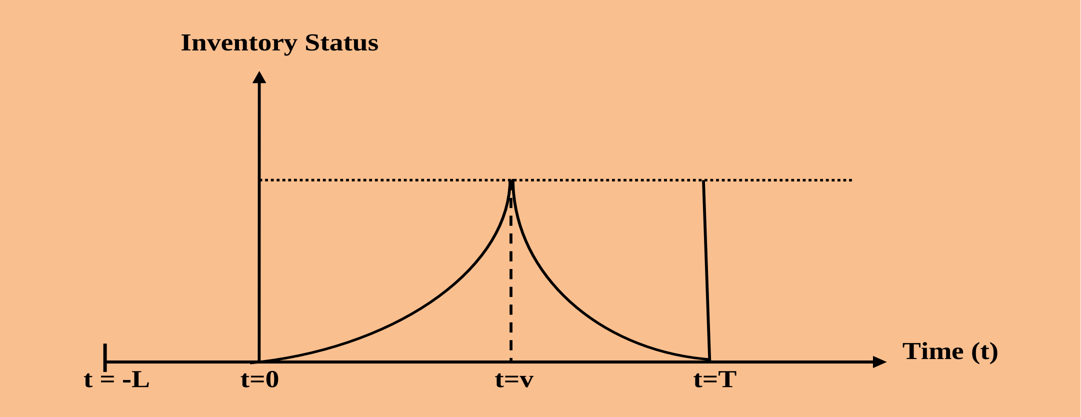


Fig.1: The graphical representation of EPQ model over time

The inventory status of the finished products at any instant of time t is governed by the following differential equations:

$$I_1'(t)+\theta I_1(t)=P-D(p), \qquad 0\le t\le v \tag{1}$$

$$I_2'(t)+\theta I_2(t)=-D(p), \qquad v\le t\le T \tag{2}$$

with the boundary condition $\mathrm{I}_1(0)=0=\mathrm{I}_2(\mathrm{T})$.

$$I_1(t)=\frac{(\lambda-1)(a-bp)}{\theta}\left(1-e^{-\theta t}\right), \qquad 0\le t\le v \tag{3}$$

$$I_2(t)=\frac{(a-bp)}{\theta}\left(e^{\theta(T-t)}-1\right), \qquad v\le t\le T \tag{4}$$

Since the inventory level, $I(t)$ is a continuous function of time, then we take $I_1(v)=I_2(v)$. After applying the continuity condition $I_1(v)=I_2(v)$, we get

$$v=\frac{1}{\theta}\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right), \tag{5}$$

The total profit of the manufacturer per cycle compromises the following components:

1. Ordering Cost: The manufacturer places an order of raw materials for production to the suppliers of raw materials at time $t=-L$. Therefore, the ordering cost per order is

$$OC=O \tag{6}$$

2. Sales Revenue: It is the income received by the manufacturer from its sales of finished goods and is calculated as

$$SR=p\left\{\int_0^T D(p)\,dt\right\}=p\,(a-bp)\,T \tag{7}$$

3. Procurement Cost: The procurement cost is the cost which manufacturer has to pay for purchasing certain raw materials from the supplier. So, the procurement cost is calculated as

$$PC=c\int_0^v I_1(t)\,dt=\frac{c(\lambda-1)(a-bp)}{\theta^2}\left(v\theta+e^{-v\theta}-1\right)$$

$$PC=\frac{c(\lambda-1)(a-bp)}{\theta^2}\left\{\log\left\{\frac{\lambda-1+e^{\theta T}}{\lambda}\right\}-\left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\} \tag{8}$$

4. Holding Cost: The holding cost per inventory cycle is

$$HC=h\left\{\int_0^v I_1(t)\,dt+\int_v^T I_2(t)dt\right\}=\frac{h(a-bp)}{\theta^2}\left\{\lambda\left(v\theta+e^{-v\theta}-1\right)+e^{\theta(T-v)}-e^{-v\theta}-\theta T\right\}$$

$$HC=\frac{h(a-bp)}{\theta^2}\left\{\lambda\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right)-\theta T\right\} \tag{9}$$

5. Deteriration Cost: The deterioration cost per inventory cycle is

$$DC=d\left\{\int_0^v I_1(t)\,dt+\int_v^T I_2(t)dt\right\}=\frac{d(a-bp)}{\theta^2}\left\{\lambda\left(v\theta+e^{-v\theta}-1\right)+e^{\theta(T-v)}-e^{-v\theta}-\theta T\right\}$$

$$DC=\frac{d(a-bp)}{\theta^2}\left\{\lambda\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right)-\theta T\right\} \tag{10}$$

6. Loan Interest: Manufacturer takes a loan from a financial institution to pay advance and cash payments for the procurement cost. So, the interest charged on the loan amount by the financial institution is calculated as

$$LI=I_c\{\alpha(L+T)+\beta T\}\times PC=\frac{cI_c(\lambda-1)(a-bp)}{\theta^2}\left(v\theta+e^{-v\theta}-1\right)\{\alpha(L+T)+\beta T\}$$

$$LI=\frac{cI_c(\lambda-1)(a-bp)}{\theta^2}\{\alpha(L+T)+\beta T\}\left\{\log\left\{\frac{\lambda-1+e^{\theta T}}{\lambda}\right\}-\left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\} \tag{11}$$

Based on the permissible delay period for the credit payments, there are three possible cases that might arise.

Case I: When $T\le M$ i.e. the permissible delay period M offered by the supplier of raw materials to the manufacturer for credit payments is greater than the inventory cycle time T.

After receiving the raw materials from the supplier, the manufacturer starts the production at time $t=0$. The manufacturer also starts the selling finished products and deposits the sales revenue in interest bearing account. During the time interval $[0, T]$, the manufacturer has sold all the produced items and earned the interest on the collected sales revenue.

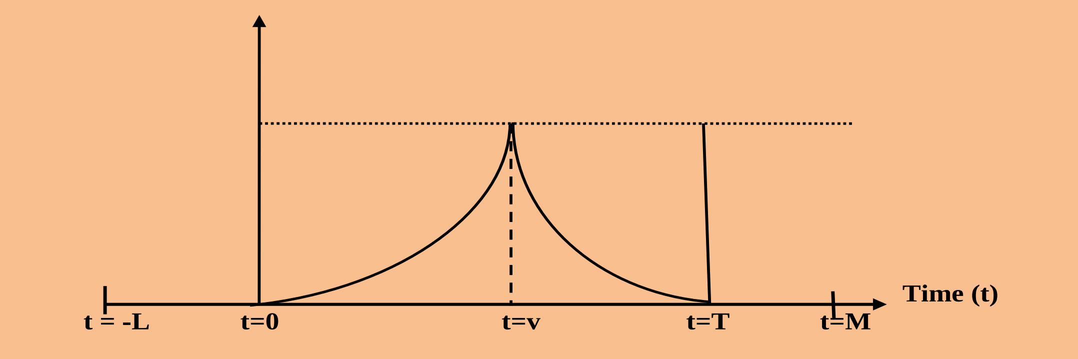


Fig.2: Case I: When $T \le M$

Therefore, the interest earned on the collected sales revenue by the manufacturer is calculated as

$$\mathrm{IE}_1[0,T] = pI_e \int_0^T D(p)\, t\, dt = \frac{1}{2} pI_e (a-bp)\, T^2 \qquad (12)$$

Total money in interest bearing account at the time $t = T$ is given by

$$\mathrm{A} = \mathrm{SR} + \mathrm{IE}_1[0,T] = \frac{p\,(a-bp)}{2}\left(2T + I_e T^2\right) \qquad (13)$$

After paying the loan amount with interest to the financial institution at the end of inventory cycle, the remaining money in the interest bearing account is

$$\mathrm{U} = \mathrm{A} - (\alpha+\beta)\times \mathrm{PC} - \mathrm{LI} = (a-bp)\left[\frac{p}{2}\left(2T + I_e T^2\right) - \frac{c(\lambda-1)}{\theta^2}\left(v\theta + e^{-v\theta} - 1\right)\left\{\begin{matrix}(\alpha+\beta)+\alpha I_c(L+T) \\ +\beta I_c T\end{matrix}\right\}\right]$$

$$U = (a-bp)\left[\frac{p}{2}\left(2T + I_e T^2\right) - \frac{c(\lambda-1)}{\theta^2}\left\{(\alpha+\beta)+\alpha I_c(L+T)+\beta I_c T\right\}\left\{\begin{matrix}\log\left(\dfrac{\lambda-1+e^{\theta T}}{\lambda}\right) \\ -\left(\dfrac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\end{matrix}\right\}\right] \qquad (14)$$

During the time interval $[T, M]$, the manufacturer also earns the interest on this remaining money. Therefore, the interest earned on this remaining money U is calculated as

$$\mathrm{IE}_1 = I_e U (M - T)$$

$$\mathrm{IE}_1 = I_e(M-T)(a-bp)\left[\frac{p}{2}\left(2T + I_e T^2\right) - \frac{c(\lambda-1)}{\theta^2}\left\{\begin{matrix}(\alpha+\beta)+\alpha I_c(L+T) \\ +\beta I_c T\end{matrix}\right\}\left\{\begin{matrix}\log\left(\dfrac{\lambda-1+e^{\theta T}}{\lambda}\right) \\ -\left(\dfrac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\end{matrix}\right\}\right]$$

(15)

The manufacturer has enough money in the interest bearing account to pay credit payments at time $t = M$. Therefore, the manufacturer has no need to pay any additional interest for credit payments to the supplier of raw materials. Thus, we have

$$\mathrm{IC}_1 = 0 \qquad (16)$$

Hence, the total profit of the manufacturer per inventory cycle is given by

$$Z_1 = \frac{1}{T}\left\{ SR - OC - PC - HC - DC - LI - IC_1 + IE_1 \right\} \tag{17}$$

Case II: When $v < M < T$ **i.e.** the permissible delay period M offered by the supplier of raw materials to the manufacturer for credit payments is less than the inventory cycle time T but greater than the production time v.

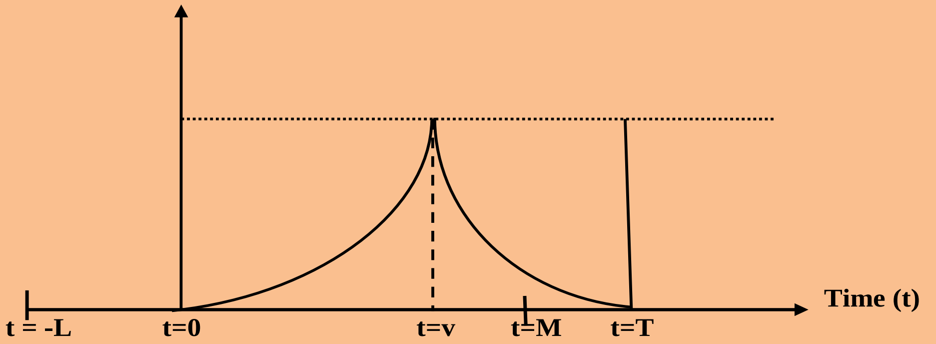


Fig.3: Case II: When $v < M < T$

After receiving the raw materials from the supplier, the manufacturer starts the production at time $t = 0$. The manufacturer also starts the selling finished products and deposits the sales revenue in interest bearing account. During the time interval $[0, M]$, the manufacturer has sold the produced items and earned the interest on the collected sales revenue. During the time period $[0, M]$, the sales revenue generated by the manufacturer is calculated as

$$SR_2[0, M] = p\int_0^M D(p)\,dt = p(a - bp)M \tag{18}$$

and the interest earned on the collected sales revenue by the manufacturer is calculated as

$$IE_2[0, M] = pI_e\int_0^M D(p)\,t\,dt = \frac{1}{2}pI_e(a - bp)M^2 \tag{19}$$

Based on the difference in the total amount in the interest bearing account of the manufacturer and the credit payments at the time $t = M$, there are two possible situations that might arise.

Subcase 2.1: When $SR_2[0, M] + IE_2[0, M] \geq \gamma PC$ **i.e.** the manufacturer has enough money in his interest bearing account to pay credit payments at time $t = M$. In this situation, the manufacturer has no need to pay any additional interest for credit payments to the supplier of raw materials. Thus, we have

$$IC_{2.1} = 0 \tag{20}$$

After paying credit payments from interest bearing account to the supplier of raw materials at the time $t = M$, manufacturer starts selling the remaining finished items and deposits this sales revenue in the same account to earn interest on it. During the time interval $[M, T]$, the interest earned on this sales revenue by the manufacturer is calculated as

$$IE_{2.1} = I_e(T - M)\{SR_2[0, M] + IE_2[0, M] - \gamma PC\} + pI_e\int_M^T D(p)\,t\,dt$$

$$IE_{2.1} = \frac{I_e(a - bp)(T - M)}{2}\left[p\begin{Bmatrix}3M + T\\ + I_eM^2\end{Bmatrix} - \frac{2c\gamma(\lambda - 1)}{\theta^2}\left\{\log\left(\frac{\lambda - 1 + e^{\theta T}}{\lambda}\right) - \left(\frac{e^{\theta T} - 1}{\lambda - 1 + e^{\theta T}}\right)\right\}\right] \tag{21}$$

In this subcase, the total profit of the manufacturer per inventory cycle is given by

$$Z_{2.1} = \frac{1}{T}\left\{ SR - OC - PC - HC - DC - LI - IC_{2.1} + IE_{2.1} \right\} \tag{22}$$

Subcase 2.2: When $\mathrm{SR}_2[0,\mathrm{M}]+\mathrm{IE}_2[0,\mathrm{M}]<\gamma\mathrm{PC}$ **i.e.** the manufacturer does not have enough money in his interest bearing account to pay credit payments at time $\mathrm{t}=\mathrm{M}$. The unpaid amount at the time of settlement is calculated as

$$\text{Unpaid Amount} =\gamma\mathrm{PC}-\{\mathrm{SR}_2[0,\mathrm{M}]+\mathrm{IE}_2[0,\mathrm{M}]\}$$

$$\text{Unpaid Amount} =\frac{(a-bp)}{2}\left[\frac{2c\gamma(\lambda-1)}{\theta^2}\left\{\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right)-\left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\}-p\{2M+I_eM^2\}\right] \tag{23}$$

Now, the supplier will allow the manufacturer to settle the unpaid amount at time $\mathrm{t}=\mathrm{T}$ and will charge interest on this unpaid amount during the time period $[\mathrm{M},\mathrm{T}]$. The interest charged on the unpaid amount by the supplier of raw materials will be

$$\mathrm{IC}_{2.2}=I_s(T-M)(\text{Unpaid Amount})$$

$$\mathrm{IC}_{2.2}=\frac{I_s(a-bp)(T-M)}{2}\left[\frac{2c\gamma(\lambda-1)}{\theta^2}\left\{\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right)-\left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\}-p\{2M+I_eM^2\}\right] \tag{24}$$

The manufacturer also sells the remaining produced items and deposits the sales revenue in the same interest bearing account to earn interest on it during the period $[\mathrm{M},\mathrm{T}]$. Therefore, the interest earned on this sales revenue is calculated as

$$\mathrm{IE}_{2.2}=pI_e\int_M^T D(p)\,t\,dt=\frac{1}{2}pI_e(a-bp)\left(T^2-M^2\right) \tag{25}$$

Hence, the total profit of the manufacturer per inventory cycle is given by

$$\mathrm{Z}_{2.2}=\frac{1}{\mathrm{T}}\{\mathrm{SR}-\mathrm{OC}-\mathrm{PC}-\mathrm{HC}-\mathrm{DC}-\mathrm{LI}-\mathrm{IC}_{2.2}+\mathrm{IE}_{2.2}\} \tag{26}$$

Case III: When $M\le v$ **i.e.** the permissible delay period M offered by the supplier of raw materials to the manufacturer for credit payments is less than the production time v.

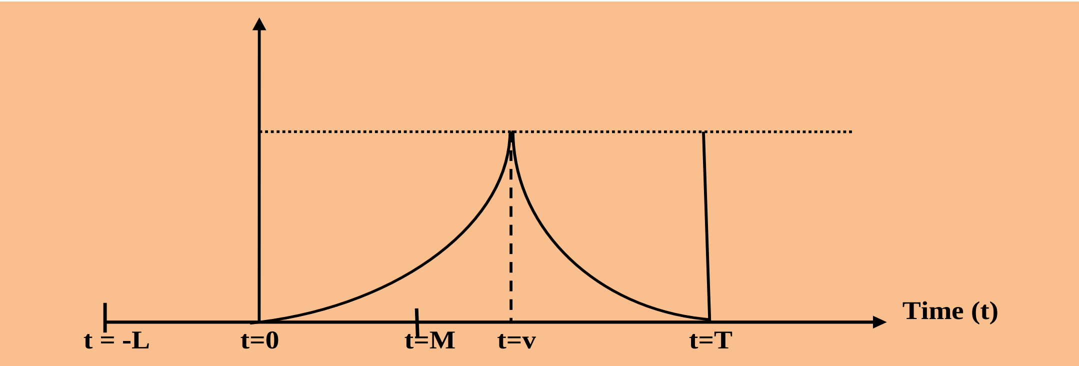


Fig.4: Case III: When $M\le v$

After receiving the raw materials from the supplier, the manufacturer starts the production at time $\mathrm{t}=0$. The manufacturer also starts the selling finished products and deposits the sales revenue in interest bearing account. During the time interval $[0,\mathrm{M}]$, the manufacturer has sold the produced items and earned the interest on the collected sales revenue. During the time period $[0,\mathrm{M}]$, the sales revenue generated by the manufacturer is calculated as

$$\mathrm{SR}_3[0,\mathrm{M}]=p\int_0^M D(p)\,dt=p(a-bp)M \tag{27}$$

and the interest earned on the collected sales revenue by the manufacturer is calculated as

$$\mathrm{IE}_3[0,\mathrm{M}] = pI_e \int_0^M D(p)\, t\, dt = \frac{1}{2} pI_e(a-bp) M^2 \tag{28}$$

Based on the difference in the total amount in the interest bearing account of the manufacturer and the credit payments at the time $\mathrm{t}=\mathrm{M}$, there are two possible situations that might arise.

Subcase 3.1: When $\mathrm{SR}_3[0,\mathrm{M}]+\mathrm{IE}_3[0,\mathrm{M}] \geq \gamma \mathrm{PC}$ **i.e.** the manufacturer has enough money in his interest bearing account to pay credit payments at time $\mathrm{t}=\mathrm{M}$. In this situation, the manufacturer has no need to pay any additional interest for credit payments to the supplier of raw materials. Thus, we have

$$\mathrm{IC}_{3.1} = 0 \tag{29}$$

After paying credit payments from interest bearing account to the supplier of raw materials at the time $\mathrm{t}=\mathrm{M}$, manufacturer starts selling the remaining finished items and deposits this sales revenue in the same account to earn interest on it. During the time interval $[\mathrm{M},\mathrm{T}]$, the interest earned on this sales revenue by the manufacturer is calculated as

$$\mathrm{IE}_{3.1} = \mathrm{I_e}(\mathrm{T}-\mathrm{M})\{\mathrm{SR}_3[0,\mathrm{M}]+\mathrm{IE}_3[0,\mathrm{M}]-\gamma\mathrm{PC}\} + \mathrm{pI_e}\int_{\mathrm{M}}^{\mathrm{T}} \mathrm{D(p)\, t\, dt}$$

$$\mathrm{IE}_{3.1} = \frac{I_e(a-bp)(T-M)}{2}\left[p\begin{Bmatrix}3M+T\\+I_eM^2\end{Bmatrix} - \frac{2c\gamma(\lambda-1)}{\theta^2}\left\{\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right) - \left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\}\right] \tag{30}$$

In this subcase, the total profit of the manufacturer per inventory cycle is given by

$$\mathrm{Z}_{3.1} = \frac{1}{\mathrm{T}}\{\mathrm{SR}-\mathrm{OC}-\mathrm{PC}-\mathrm{HC}-\mathrm{DC}-\mathrm{LI}-\mathrm{IC}_{3.1}+\mathrm{IE}_{3.1}\} \tag{31}$$

Subcase 3.2: When $\mathrm{SR}_3[0,\mathrm{M}]+\mathrm{IE}_3[0,\mathrm{M}] < \gamma \mathrm{PC}$ **i.e.** the manufacturer does not have enough money in his interest bearing account to pay credit payments at time $\mathrm{t}=\mathrm{M}$. The unpaid amount at the time of settlement is calculated as

$$\text{Unpaid Amount} = \gamma\mathrm{PC} - \{\mathrm{SR}_3[0,\mathrm{M}]+\mathrm{IE}_3[0,\mathrm{M}]\}$$

$$\text{Unpaid Amount} = \frac{(a-bp)}{2}\left[\frac{2c\gamma(\lambda-1)}{\theta^2}\left\{\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right) - \left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\} - p\{2M+I_eM^2\}\right] \tag{32}$$

Now, the supplier will allow the manufacturer to settle the unpaid amount at time $\mathrm{t}=\mathrm{T}$ and will charge interest on this unpaid amount during the time period $[\mathrm{M},\mathrm{T}]$. The interest charged on the unpaid amount by the supplier of raw materials will be

$$\mathrm{IC}_{3.2} = I_s(T-M)(\text{Unpaid Amount})$$

$$\mathrm{IC}_{3.2} = \frac{I_s(a-bp)(T-M)}{2}\left[\frac{2c\gamma(\lambda-1)}{\theta^2}\left\{\log\left(\frac{\lambda-1+e^{\theta T}}{\lambda}\right) - \left(\frac{e^{\theta T}-1}{\lambda-1+e^{\theta T}}\right)\right\} - p\{2M+I_eM^2\}\right] \tag{33}$$

The manufacturer also sells the remaining produced items and deposits the sales revenue in the same interest bearing account to earn interest on it during the period $[\mathrm{M},\mathrm{T}]$. Therefore, the interest earned on this sales revenue is calculated as

$$\mathrm{IE}_{3.2} = pI_e\int_M^T D(p)\, t dt = \frac{1}{2} pI_e(a-bp)(T^2-M^2) \tag{34}$$

Hence, the total profit of the manufacturer per inventory cycle is given by

$$\mathrm{Z}_{3.2} = \frac{1}{\mathrm{T}}\{\mathrm{SR}-\mathrm{OC}-\mathrm{PC}-\mathrm{HC}-\mathrm{DC}-\mathrm{LI}-\mathrm{IC}_{3.2}+\mathrm{IE}_{3.2}\} \tag{35}$$

4. Numerical Analysis of the EPQ Model

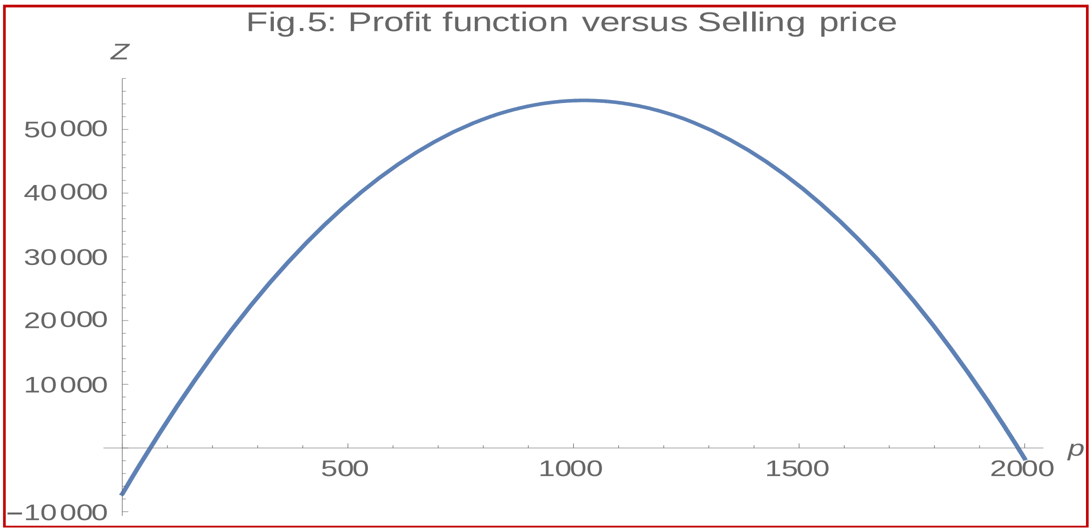
The main purpose of numerical analysis of this EPQ inventory model is to find the optimal selling price of the finished product and to test the concavity of the profit function of the manufacturer per inventory cycle with respect to the selling price.

Example1: Let $a = 100$, $b = 0.05$, $\theta = 0.01, \lambda = 2$, $O = \text{Rs } 5000$ per order, $c = \text{Rs } 100$ per unit, $h = \text{Rs } 2$ per unit per year, $d = \text{Rs } 1.5$ per unit per year, $I_e = 8\,\%$ per rupee per year, $I_s = 12\,\%$ per rupee per year, $I_c = 14\,\%$ per rupee per year, $\alpha = 0.25$, $\beta = 0.30, \gamma = 0.45$, $L = 1$ year, $T = 3$ years and $M = 5$ years.

With the help of the Mathemetica 11.2, we obtain the following optimum results.

$$p^* = 1023.54 \text{ per unit}, v^* = 1.51125 \text{ years, and } Z^* = \text{Rs } 54550.43$$

Plot of the profit function of the manufacturer per inventory cycle with respect to the selling price for case when $T \leq M$ is shown in Fig.5.



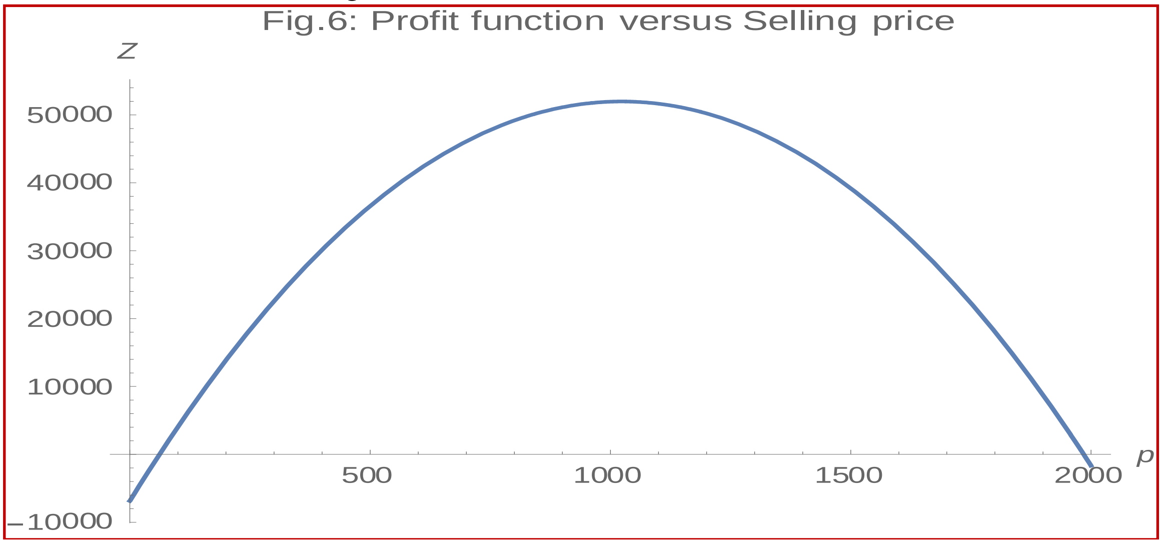
From the figure 5, it is clear that the profit function (Z) of the manufacturer per inventory cycle is a concave function of the selling price (p) for case when $T \leq M$.

Example2: Let $a = 100$, $b = 0.05$, $\theta = 0.01, \lambda = 2$, $O = \text{Rs } 5000$ per order, $c = \text{Rs } 100$ per unit, $h = \text{Rs } 2$ per unit per year, $d = \text{Rs } 1.5$ per unit per year, $I_e = 8\,\%$ per rupee per year, $I_s = 12\,\%$ per rupee per year, $I_c = 14\,\%$ per rupee per year, $\alpha = 0.25$, $\beta = 0.30, \gamma = 0.45$, $L = 1$ year, $T = 3$ years and $M = 2$ years.

With the help of the Mathemetica 11.2, we obtain the following optimum results.

$$p^* = 1023.09 \text{ per unit}, v^* = 1.51125 \text{ years, and } Z^* = \text{Rs } 51979.84$$

Plot of the profit function of the manufacturer per inventory cycle with respect to the selling price for case when $v < M < T$ is shown in Fig.6.



From the figure 6, it is clear that the profit function (Z) of the manufacturer per inventory cycle is a concave function of the selling price (p) for case when $v < M < T$.

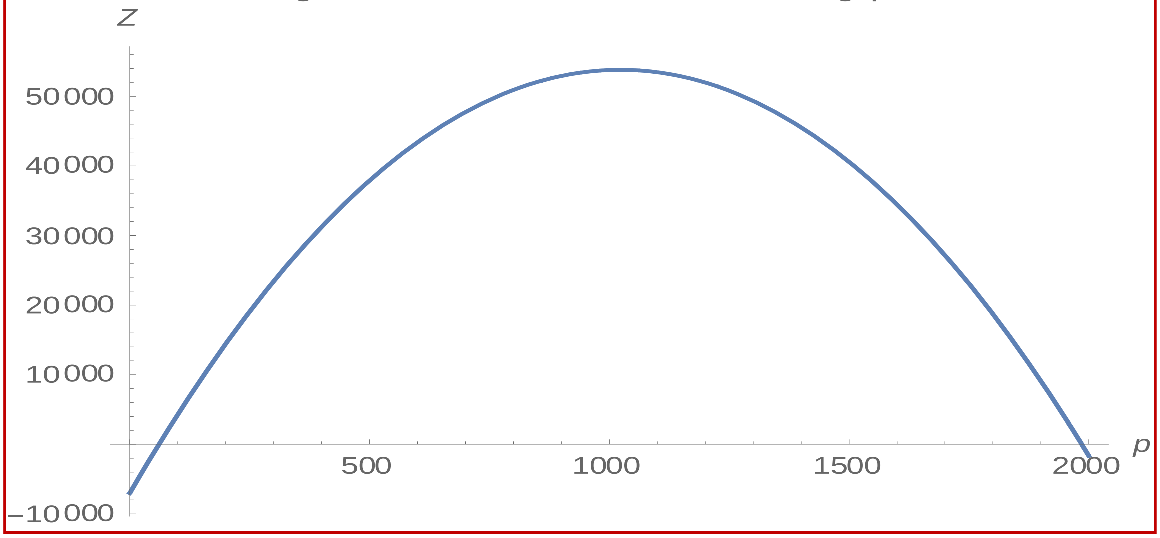
Example3: Let $a = 100$, $b = 0.05$, $\theta = 0.01$, $\lambda = 2$, $O = \text{Rs } 5000$ per order, $c = \text{Rs } 100$ per unit, $h = \text{Rs } 2$ per unit per year, $d = \text{Rs } 1.5$ per unit per year, $I_e = 8\%$ per rupee per year, $I_s = 12\%$ per rupee per year, $I_c = 14\%$ per rupee per year, $\alpha = 0.25$, $\beta = 0.30$, $\gamma = 0.45$, $L = 1$ year, $T = 3$ years and $M = 1$ years.

With the help of the Mathemetica 11.2, we obtain the following optimum results.

$$p^* = 1022.93 \text{ per unit}, \; v^* = 1.51125 \text{ years, and } Z^* = \text{Rs } 53805.57$$

Plot of the profit function of the manufacturer per inventory cycle with respect to the selling price for case when $M \le v$ is shown in Fig.7.

Fig.7: Profit function versus Selling price



From the figure 7, it is clear that the profit function (Z) of the manufacturer per inventory cycle is a concave function of the selling price (p) for case when $M \le v$.

5. Conclusion

In our present article, we have formulated an EPQ model for deteriorating product. It is assumed that the rate of production of the finished products depends on the demand and the manufacturer follows the advance-cash-credit (ACC) payment policy to pay for its purchasing of raw materials. For the proposed EPQ model, the concavity of the profit functions have been demonstrated graphically with the help of the numerical examples. This EPQ model has the potential to be extended to include inflation and quantity discount effects, different demand patterns such as time dependent demand, and other issues under the system with or without shortages.

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